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Understanding the Keynesian view of Business Cycles
- A Dynamic AS-AD Model

Second edition 2008
(A few typos corrected 2010)

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1 Introduction

This text gives a short description of the theory beyond the dynamic AS-AD model. For a more rigours treatment of the theory beyond the AS and AD curve se any textbook in macroeconomics. This text comes together with an excel sheet where the students can play with the model and se how the economy will react for different kind of disturbances under different structural assumption.[1]

The AS-AD model is based upon a general equilibrium where the markets in the economy are divided into three categories. The three markets are the market for currency where the exchange rate is determined, the market for goods and services where the inflation rate is determined and the market for money where the interest rate is determined. All these markets interact. One example of this interaction is that a change in the money market gives a decrease in interest rate that will lead to increased investment and thus induce a change in the market for goods and services. The interest rate will also have an impact on the exchange rate and the exchange rate have an impact on net trade and thus on the market for goods and services. A general equilibrium is a situation where all the three market are in equilibrium at the same time.

In neoclassical models this equilibrium will always occur at the equilibrium level of unemployment. Keynesian models assume that the equilibrium may occur at a level of unemployment that is higher or lower than the equilibrium level and business cycles are viewed as cyclical deviations from the long run equilibrium. Neoclassical models, like the real business cycles, look upon business cycles as cyclical movement of the equilibrium levels. The AS-AD model described here is an example of a Keynesian model. It can however also be calibrated from the neoclassical assumption of rational expectations. But in that case no business cycles will occur since productivity chocks, the driving forces of business cycles in the RBC theory, are not included in the model. In most textbooks in macroeconomics, open economy aggregate demand curves are derived from the Mundel Flemming model. In this text I will go straight to the AS-AD model but still explains the mechanisms underlying it.

The second edition of this text includes a little bit more of the theory beyond the model. Moreover some development of the model is made, the government deficits and automatic stabilizers are modelled more consistent. In the previous edition the impact of automatic stabilizers where not included in the calculation of the government deficit. Moreover it is possible to model other demand disturbances than fiscal and monetary policy. In the excel sheet another indata column is added called “Other demand chocks”.

The model consists of the aggregate supply and aggregate demand curves. The values of inflation and output gap in this time period will determine how the short run curves shift to the next period. These movements will make the economy move in loops around the long run equilibrium, the intersection between the long run aggregate demand and aggregate supply curves. The size of the movements will for example depend upon how the actors in the economy form their expectations and whether politicians let the automatic stabilizers work or adjust current expenditure to current tax income.

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[1] The excel model could be downloaded from www.natskolan.se/exercises

The next section will explain the theory beyond the aggregate supply curves and section three the theory beyond the aggregate demand curves. In section four aggregate demand and aggregate supply are put together and the behaviour over time of the model is explained. The mathematical derivation of the solution of the model is given in the appendix. The equations derived there are used in the excel sheet.

2 Aggregate supply

The output gap, ***y*** is defined as the deviation from long run equilibrium output. Thus the long run aggregate supply curve simply becomes:

$$y = 0 \qquad 1$$

The short run aggregate supply curve AS is assumed to have the following form;

$$\pi_t = \overline{\pi}_t + b \cdot y_t + s_t \qquad 2$$

where ***π*** is inflation, ***s*** is supply chocks and ***b*** is a coefficient determine how sensible inflation is to changes in the output gap.

It says that inflation in period ***t*** will depend upon underlying inflation, the output gap and supply chocks. Actual inflation is the result of wage negotiations and price setting. Wage negotiations occur the time period before period one. Workers want compensation for price increases that have occurred in the period before, i.e period ***t-1***. But they also want to have compensation for price increases that they expect to occur in the following period i.e period ***t***. And a similar reasoning will apply for prices set by firms that publish their price lists for a specific period in advance.

Underlying inflation will thus depend upon two things, last period's inflation and expectations about this period's inflation at the time when prices and wages are set.

$$\overline{\pi}_t = f\left(\pi_{t-1}, E(\pi_t)\right) \qquad 3$$

The crucial thing here is what determines expectations about future inflation. One possibility is perfect foresight. In that case expectations would be equal to actual inflation i.e.

$$E(\pi_t) = \pi_t \qquad 4$$

Another possibility is that expectations are adaptive. That means that people expect the inflation rate next year to be the same as it was this year.

$$E(\pi_t) = \pi_{t-1} \qquad 5$$

Underlying inflation could be written as;

$$\overline{\pi}_t = \lambda \pi_t + (1-\lambda)\pi_{t-1} \qquad 6$$

Where the value of lambda defines how the expectations are formed[2]. Note that if all actors in the economy have adaptive expectation the λ in equation 6 will be equal to zero since in that case underlying inflation will be equal to last period's inflation. If all actors in the economy have perfect foresight the value of λ will be a fairly large number. It will still not be equal to one though, since past inflation always is a part of underlying inflation. Remember that underlying inflation consists both of expectations about future inflation and claims of compensation for last period's inflation. Experiments about the role of expectations could be made by changing the value of λ in the simulations.

Perfect foresight is often referred to as rational expectations. This terminology may be misleading since making very advanced forecast about future inflation may be costly. Assuming inflation to be the same as it was this year, may be a cost effective behaviour of price setters and thus pretty rational. If inflation is rather stable over time this year inflation will most often be a very good prediction of next year inflation and thus a rather good guess. So defining a rational behaviour of price setters is not an easy task. However in the following the term rational expectations will be used for actors that, at least on average, tend to make accurate guesses about the future.

To derive the AS curve in terms of actual inflation, inserting equation 6 into equation 2;

$$\pi_t = \lambda\pi_t + (1-\lambda)\pi_{t-1} + b \cdot y_t + s_t \qquad 7$$

subtract $\boldsymbol{\lambda\pi_t}$ from both sides;

$$(1-\lambda)\pi_t = (1-\lambda)\pi_{t-1} + b \cdot y_t + s_t \qquad 8$$

and divide by ***1- λ***.

$$\pi_t = \pi_{t-1} + \frac{b}{(1-\lambda)} \cdot y_t + \frac{1}{(1-\lambda)} s_t \qquad 9$$

Equation 9 is the AS curve solved for the rate of inflation at time t.

[2] Lambda will also reflect the share of long term price and wage contracts in the economy. With a high proportion of long term contracts past prices will be more sluggish and lambda will be lower.

3 Aggregate demand

Assume aggregate demand to be a function of the growth rate of real money supply and demand chocks like fiscal policy or changes household consumption. Since real money supply is equal to M/P the change in real money supply would be equal to the growth rate of nominal money supply less the inflation rate. If inflation is higher then the growth rate of nominal money supply, real money supply will decrease. The short run AD curve will thus be equal to:

$$y_t = a_2 \cdot (\mu_t - \pi_t) + a_3 \cdot d_t \qquad 10$$

Where a_2 is a coefficient that determines the sensitivity of demand to changes in real money supply, μ is nominal money growth, d is demand disturbances other than changes in real money supply and a_3 is a coefficient that determines the sensitivity of these demand disturbances on aggregate demand.

Note that with flexible exchange rate the value of a_3 will be fairly small. In the simplest versions of the Mundel Flemming model where interest rates are the same all over the world a_3 would be equal to zero. In reality differences in interest rates could be explained from expectations about future changes of the exchange rate and demand disturbances may have an impact on the AD curve even under flexible exchange rate even if that effect is smaller than under a fixed exchange rate regime.

An increase in the interest rate in one country would increase the demand of the currency. The currency would appreciate but that appreciation will create expectations of future depreciations back to the equilibrium level. Thus the interest rate will not fall back to the international level but only to a level where the difference in interest rates compensate for the expected depreciation.

Note also that under a fixed exchange rate regime the nominal money supply will be determined from the amount of liquidity needed to keep the exchange rate at its parity. In this model we will thus assume a flexible exchange rate so that we are able to use monetary policy to stabilize demand.

There are some good reasons to believe consumption and investments to be functions of last year's output, at least if expectations are adaptive. In that case increases in sales are assumed to be permanent and will thus gives incentives to increase investments in order to increase production possibilities. But since it takes some time between planning and implementation of investments these higher investments will probably not occur until next period. Increases in income will increase household's consumption but there may take some time before households change their behaviour. Adding this to our AD curve it will become;

$$y_t = a_1 \cdot y_{t-1} + a_2 \cdot (\mu_t - \pi_t) + a_3 \cdot d_t \qquad 11$$

where a_1 will depend upon the kind of expectations. With perfect foresight (rational expectations) firms will know the future selling and base their investments decision on actual output in next period giving a low value of a_1.

In the long run inflation is determined from nominal money growth and the long run aggregate demand curve LAD is

$$\pi = \mu \qquad 12$$

Fiscal Policy, Automatic stabilizers and other demand chocks

In recessions income taxes decreases and transfers such as unemployment's benefits to households increases. In booms the opposite occur i.e. taxes increases and transfers decreases. This is often called automatic stabilizers since it make fiscal policy expansionary in recessions and tight in the booms even if no decisions are made by the government. Fiscal policy would thus be equal to the sum of active fiscal policy and the impact from automatic stabilizers. And total demand disturbances would be equal to this sum plus other demand disturbances.

$$d_t = f_t + a_4(y_{t-1} - y_{t-2}) + o_t \qquad 13$$

In equation 13 **f** is active fiscal policy, $\boldsymbol{a_4}$ is the sensitivity of the governmental budget to changes in output of previous period and ***o*** other demand disturbances. The logic for using previous period's growth is that firms may not be able to adjust their working force immediately after a fall in sales and the effect on unemployment's benefits and income taxes may thus come with a lag.

The theory of automatic stabilizers says that $\boldsymbol{a_4}$ should be negative and fiscal policy countercyclical. But the value of $\boldsymbol{a_4}$ would depend on the behaviour of politicians. If politicians respond to a boom by increasing spending or cut taxes fiscal policy will be pro cyclical. If this effect is large the value of $\boldsymbol{a_4}$ will come close to zero.

Our AS-AD model consists of equation 9, 11 and 13. In the file ASAD.xls you can run simulations with this model. For the derivation of the equations used in the excel sheet, see appendix 1

4 The behaviour over time of the dynamic AS-AD model

Mathematical explanation

Consider first the AD curve (equation 11) and assume that there are no demand disturbances i.e. ***d = 0*** in all time periods and that $a_1 = 1$ so that the AD curve becomes;

$$y_t = 1 \cdot y_{t-1} + a_2 \cdot (\mu_t - \pi_t) \qquad 14$$

Consider the following three cases:

$$\mu_t = \pi_t$$

In this case the AD curve says the output gap this period will be the same as the output gap in previous period since the second term will be equal to zero. It is only when inflation is equal to nominal money growth that there is no horizontal movements in the diagram. That is when the economy is on the long run aggregate demand curve.

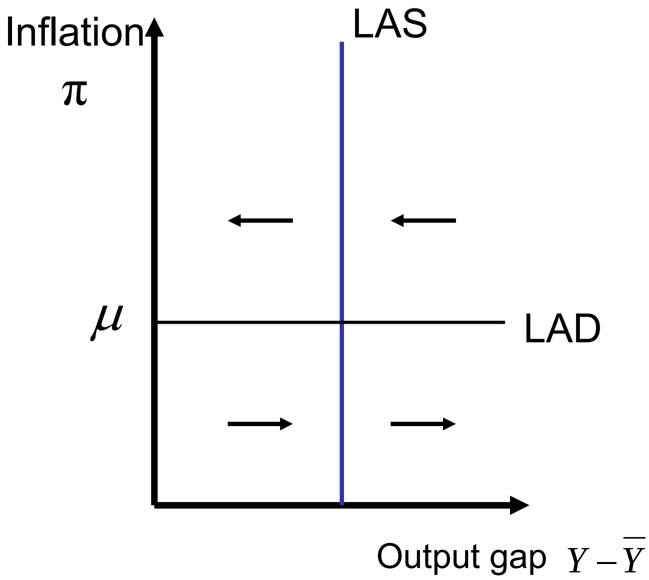
$$\pi_t > \mu_t$$

In this case, when the economy is above LAD, the second term will be negative and the output gap will decrease. In the graphical AS-AD diagram the economy will move to the left. The reason for this movement is that if inflation is higher than nominal money growth, real money supply will decrease and thus reducing aggregate demand.

$$\pi_t < \mu_t$$

In this case the second term will be positive and the output gap will increase and the economy will move to the right. Real money stock increases and thus demands increases. These findings are summarized in figure 1, showing in what direction the economy will move on different sides of LAD.

Figure 1 Horizontal movements in the AS-AD model



Consider now the AS curve, equation 9. Assume that there are no supply chocks so that the last term is equal to 0, so that the AS curve becomes;

$$\pi_t = \pi_{t-1} + \frac{b}{(1-\lambda)} \cdot y_t \qquad 15$$

Consider the following three cases:

$$y_t = 0$$

In this case the AS curve says the inflation in this period will be the same as the inflation in previous period since the second term will be equal to zero. It is only when the output gap is equal to zero that there is no vertical movements in the diagram. That is, when the economy is on the long run aggregate supply curve and has a non accelerating inflation rate of unemployment.

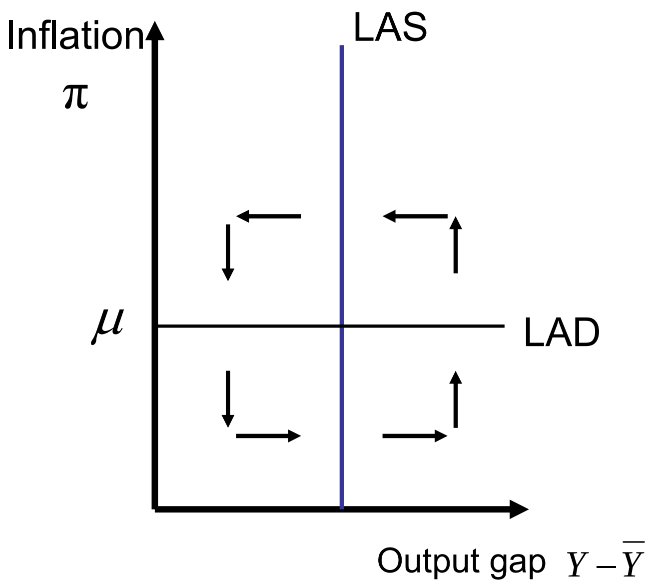
$$y_t > 0$$

In this case the second term will be positive and the inflation will increase. In the graphical ASAD diagram the economy will move upwards. When the output gap is positive we want to purchase more commodities than the economy is able to produce and it will be a pressure upwards on prices.

$$y_t < 0$$

In this case the second term will be negative and the inflation will decrease and the economy will move downwards. The AS curve always crosses the long run aggregate supply at the level of underlying inflation. If we are to the left of LAS inflation always turns out to be lower than expected and thus expectations decrease even further so that underlying inflation falls to the next period and the short run AS curve shifts downwards. Figure 2 shows the movement of the economy on different sides of LAS as well as the previously defined horizontal movements.

Figure 2 Horizontal and vertical movement in the AS-AD model



It can be seen that if the economy for some reason gets off the long run equilibrium, i.e the intersection between the long run aggregate demand and aggregate supply curves, it will get a circular movement around the long run equilibrium.

Verbal explanation

What is the intuition beyond the development in the four different parts of the loop when we do not have any demand or supply chocks? Consider first the upper right part. Here inflation is higher than nominal money growth and the output gap is positive. A positive output gap means that aggregate demand is higher than long run aggregate supply. There will be lack of goods and it will be easier for firms to increase their prices. Thus inflation will increase. Another explanation, that is a little bit more mathematical, is that the AS curve always cross LAS at the level of underlying inflation. So if the output gap is positive actual inflation is always higher than expected. With adaptive expectation the actors in the economy will increase their expectation of inflation to the next period meaning that the short run AS curve will shift upwards. This is the intuition beyond the movement upwards in the figure.

Since inflation is higher than nominal money growth real money supply will decrease. Thus interest rates will increase. This effect is even stronger since the positive output gap gives a demand for money that is higher than average. The increase in interest rates give rise to an inflow of financial capital that makes the increase smaller that it would have been in a closed economy. But that inflow appreciates the exchange rate. Since inflation is higher than normal the real exchange rate will appreciate even more than the nominal exchange rate. The appreciation of the real exchange rate decreases net export and the increase in interest rates decreases investments. Thus aggregate demand decreases and so does the output gap. This explains the movement to the left in the figure.

How large will the increase in interest rates be? The higher than normal rate of inflation should probably give rise to expectations about a lower nominal exchange rate in the future in order to hold real exchange rate constant. Together with the initial appreciation this creates an expectation of future depreciation of the nominal exchange rate that must be compensated for by an increase in the interest rate for the interest rate parity to hold.

Consider now the upper left quadrant. Here the output gap is negative, aggregate demand is lower than long run aggregate supply. It is difficult for firms to sell their products and they cannot increase prises that much. Thus inflation tends to decrease. When we are to the left of LAS actual inflations is lower than expected inflation, with adaptive expectations the actors in the economy adjust their expectations and the AS curve shifts downward. Since we are above LAD inflation is higher than nominal money growth decreasing real money supply. The upward pressure on the interest rate is not that strong though since the negative output gap gives a lower demand for money. From the same logic as in the upper right quadrant increased interest rate reduces aggregate demand explaining the movement to the left.

In the lower left quadrant output gap is negative and there is a pressure downwards on inflation just as in the upper left quadrant explaining the movement downwards. Here inflation is lower then nominal money growth. Real money supply is increasing and interest rates are decreasing. The negative output gap gives a demand for money that is lower than average giving an even higher downward pressure on the interest rate. The decrease in interest rates give rise to an outflow of financial capital that makes the decrease smaller that it would have been in a closed economy. But that outflow depreciates the exchange rate. Since inflation is lower than normal the real exchange rate will depreciate even more than the nominal exchange rate. The depreciation of the real exchange rate increases net export and the decrease in interest rates increases investments. Thus aggregate demand increases and so does the output gap. This explains the movement to the right in the figure.

In the lower right quadrant output gap is positive giving a pressure upward on inflation explaining the movement upwards. Inflation is lower than nominal money growth giving a pressure downwards in interest rate even if it is not as strong as in the lower left quadrant since the demand for money increases with the positive output gap. The decrease in interest rates increases aggregate demand following the same logic as in the lower left quadrant explaining the movement to the right.

Will the economy reach the long run equilibrium?

Up until now it has been assumed that $a_1 = 1$. Assume now instead that $a_1 < 1$. This will give the economy an additional force from the first term of the AD curve (equation 11). If last years output gap was positive this year's output gap will still be positive but have a lower value since it is multiplied by a number less than one. If last years output gap was negative this year output gap will be negative but have a negative number more close to zero. The result is that the economy will move closer and closer to the long run equilibrium over time. If a_1 is large the economy will instead move away from the equilibrium.

However there is more kind of forces that drag the economy towards equilibrium so even with $a_1 > 1$ the economy may end up in the long run equilibrium depending on the values of ***b***, ***λ***, a_2 and a_4. The values of all these parameters will determine whether the circles will become smaller or larger over time, i.e. whether the system will go towards equilibrium or whether it will explode. For most realistic values of the parameters the system will move towards equilibrium.

A decrease of the long run growth of nominal money supply

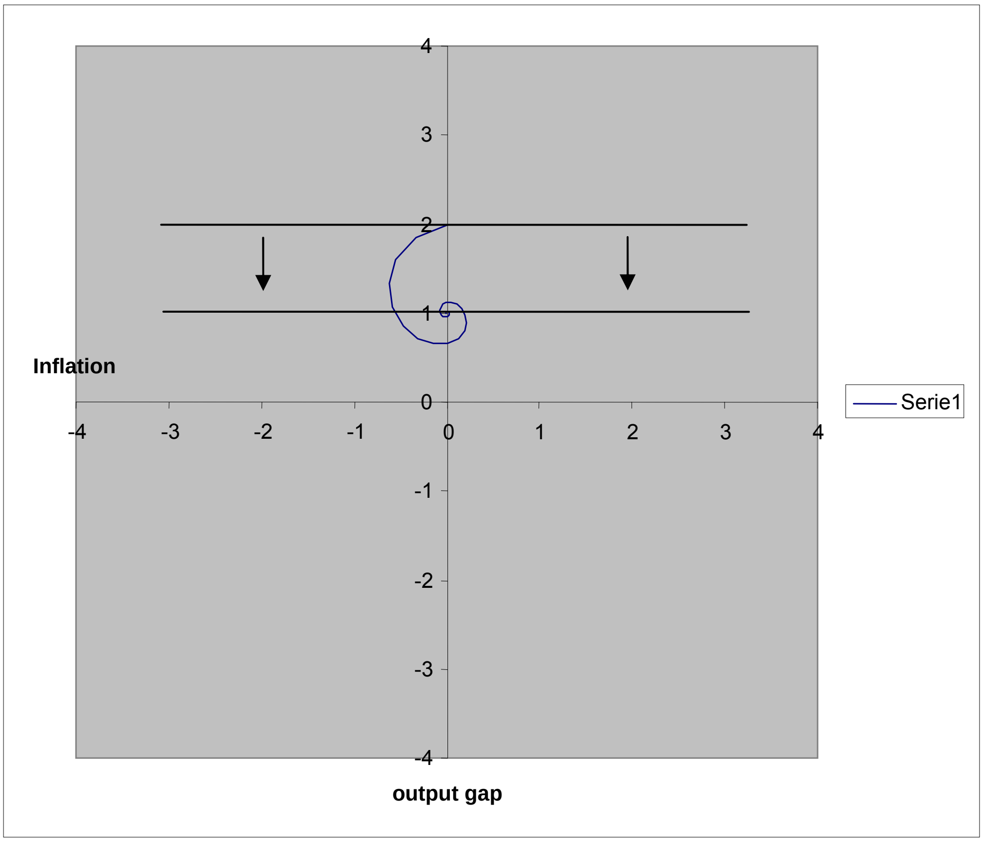
As an example of how the model could be used we will simulate the effect from a permanent decrease of the long run growth in nominal money supply i.e. a downward shift in the LAD curve. To check how the model work we plug in some number into ASAD.xls. In the simulation below the following assumption where used;

The effect of real money growth on the output gap, a_2 is assumed to be 0.4 giving a sloop of the short run AD curve of -0.4 (since inflation have a negative impact on real money growth). The effect of the output gap on inflation, ***b*** is also assumed to be 0.4 giving a sloop of the short run AS curve of 0.4.

Expectations are assumed to be adaptive plugging in a low value of ***λ*** and a high value of a_1. Assuming no automatic stabilizers the value of a_4 is equal to zero. The politicians in this country always react to booms by increasing expenditure and cut expenditure when a recession is expected and thus the effect from unemployment benefits and income taxes on the government budget is neutralized. Assume a flexible exchange rate and thus small effects from demand disturbances so that a_3 have a small value.

How will this model react to a permanent decrease in the nominal money growth? Assume the economy had been in long run equilibrium with a nominal money growth of 2 percent a year that is suddenly changed to 1 percent per year. The immediate effect is that the long run aggregate demand curve falls since it is always equal to the growth rate of nominal money supply.

Figure 3 LAD shifts downwards



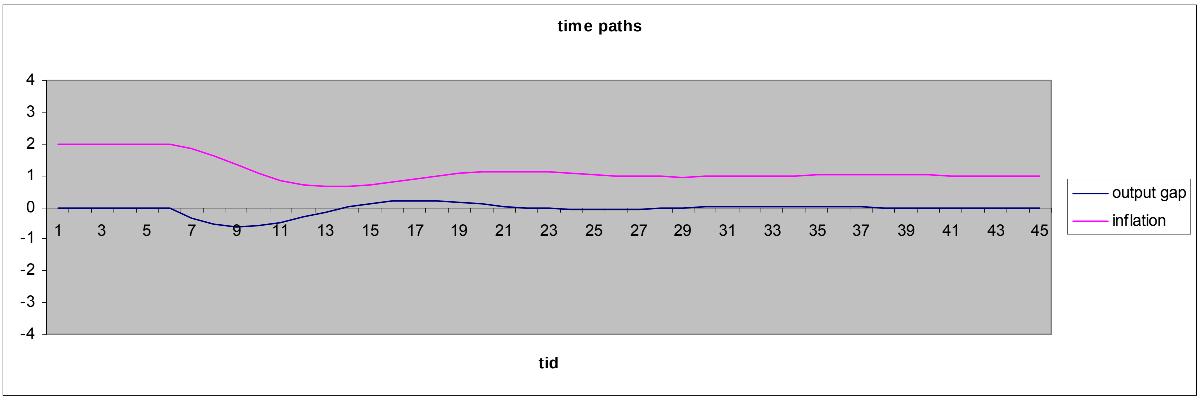
Parameters used	
λ	0,1
a1	0,9
a2	0,4
a3	0,1
a4	0
b	0,4

When the growth rate of nominal money falls and inflation is still high real money supply decreases. When real money supply decreases the interest rate will increase. The increase in interest rate attracts foreign investors giving an appreciation of the exchange rate and a pressure downwards again on the interest rate. This appreciation will be so large that it will overshoot the long run level of the exchange rate. Thus we get an expected depreciation large enough to compensate for the higher domestic interest rate. Since the increase in interest rate decreases investment and the appreciation of the exchange rate decreases exports aggregate demand will decrease and the economy moves leftwards.

This movement leftwards will continue as long as inflation is above nominal money growth i.e. as long as we are above the LAD curve. When the output gap becomes more negative, inflation starts to fall faster and faster since firms have difficulties in selling their products and thus tend to set lower prices. When inflation falls it reduces underlying inflation, i.e the AS curve shifts downward over time. However when inflation has become lower than nominal money growth, i. e. when the economy is below the long run aggregate demand curve, real money starts to grow and the economy move to the right. When the output gap turns positive and aggregate demand is higher than the long run aggregate supply, inflation starts to grow again, and with it underlying inflation, shifting the AS curve upwards. To summarize the economy moves towards the new long run equilibrium in a loop around it.

The time paths of inflation and output gap are shown in the figure 4:

Figure 4 LAD shifts downwards



It can be seen that the reduction of nominal money growth gives a series of business cycles before the economy adjust to the lower rate of inflation consistent with the new long run aggregate demand curve. The output gap starts growing in each cycle before inflation stops falling. First when the output gap becomes positive inflation starts to grow. When inflation is above the nominal money growth output start falling. But inflation continues to grow until output gap turns negative and starts dampen inflation. The decrease in the long run aggregate demand curve creates a series of smaller and smaller business cycles where changes of the output gap always occur before the changes in inflation.

Exercise:

Plug these number into the excel model and se what happens if you change some of the parameters. Try for example to increase the value of λ and / or decrease the value of a_1.

Appendix 1

The AS-AD model consists of equation 9 and 11. The intersection between these two curves gives the inflation rate and the output gap of the economy. In this appendix the solution two this equation system will be derived.

To get the change in the output gap from one year to the other the lagged value of equation 11 is subtracted on both left and right hand side of equation 11.

$$y_t - y_{t-1} = a_1 \cdot y_{t-1} - (a_1) \cdot y_{t-2} + a_2 \cdot (\mu_t - \mu_{t-1}) - a_2 \cdot (\pi_t - \pi_{t-1}) + a_3 \cdot d_t - a_3 \cdot d_{t-1} \quad \text{A1}$$

Subtract last year inflation from both sides of equation 9 gives an expression for the change in inflation.

$$\pi_t - \pi_{t-1} = \frac{b}{(1-\lambda)} \cdot y_t + \frac{1}{(1-\lambda)} s_t \quad \text{A2}$$

Inserting equation A2 in equation A1 gives us the intersection between the AS and AD curve;

$$y_t - y_{t-1} = a_1 \cdot y_{t-1} - (a_1) \cdot y_{t-2} + a_2 \cdot (\mu_t - \mu_{t-1}) - a_2 \cdot \left(\frac{b}{(1-\lambda)} \cdot y_t + \frac{1}{(1-\lambda)} s_t \right) + a_3 \cdot d_t - a_3 \cdot d_{t-1} \quad \text{A3}$$

Simplifies to;

$$y_t = (1 + a_1) \cdot y_{t-1} - (a_1) \cdot y_{t-2} + a_2 \cdot (\mu_t - \mu_{t-1}) - a_2 \cdot \frac{b}{(1-\lambda)} \cdot y_t - a_2 \cdot \frac{1}{(1-\lambda)} s_t + a_3 \cdot d_t - a_3 \cdot d_{t-1} \quad \text{A4}$$

$$\left(1 + \frac{a_2 \cdot b}{(1-\lambda)} \right) y_t = (1 + a_1) \cdot y_{t-1} - (a_1) \cdot y_{t-2} + a_2 \cdot (\mu_t - \mu_{t-1}) - \frac{a_2}{(1-\lambda)} s_t + a_3 \cdot d_t - a_3 \cdot d_{t-1} \quad \text{A5}$$

$$y_t = \alpha_1 \cdot y_{t-1} - \alpha_2 \cdot y_{t-2} + \beta_1 \cdot \Delta\mu_t - \beta_2 \cdot s_t + \beta_3 \cdot \Delta d_t \qquad \text{A6}$$

Where;

$$\alpha_1 = \frac{(1+a_1)}{\left(1+\frac{a_2 \cdot b}{(1-\lambda)}\right)} \qquad \alpha_2 = \frac{a_1}{\left(1+\frac{a_2 \cdot b}{(1-\lambda)}\right)}$$

$$\beta_1 = \frac{a_2}{\left(1+\frac{a_2 \cdot b}{(1-\lambda)}\right)} \qquad \beta_2 = \frac{\frac{a_2}{(1-\lambda)}}{\left(1+\frac{a_2 \cdot b}{(1-\lambda)}\right)} \qquad \beta_3 = \frac{a_3}{\left(1+\frac{a_2 \cdot b}{(1-\lambda)}\right)}$$

Equation A6 gives an expression for the development over time of the output gap that takes both the AS and the AD curve into account. But an equation for inflation is needed in the excel model as well. Using equation A2 and solving for the output gap gives.

$$y_t = \frac{1-\lambda}{b}(\pi_t - \pi_{t-1}) - \frac{1}{b} s_t \qquad \text{A7}$$

substituting A7 and its lag into equation 11 gives ones again the intersection between the AS and AD curve but this time it will give us the value of inflation in that point.

$$\frac{1-\lambda}{b}(\pi_t - \pi_{t-1}) - \frac{1}{b} s_t = $$
$$(a_1) \cdot \left(\frac{1-\lambda}{b}(\pi_{t-1} - \pi_{t-2}) - \frac{1}{b} s_{t-1}\right) + a_2 \cdot (\mu_t - \pi_t) + a_3 \cdot d_t \qquad \text{A8}$$

that simplifies to;

$$(1-\lambda) \cdot \pi_t - (1-\lambda) \cdot \pi_{t-1} - s_t = (a_1) \cdot ((1-\lambda) \cdot \pi_{t-1} - (1-\lambda) \cdot \pi_{t-2} - s_{t-1})$$
$$+ b \cdot a_2 \cdot \mu_t - b \cdot a_2 \cdot \pi_t + b \cdot a_3 \cdot d_t \qquad \text{A9}$$

$$\left(1-\lambda+b\cdot a_2\right)\cdot\pi_t=\left(1+a_1\right)\cdot\left(1-\lambda\right)\cdot\pi_{t-1}-$$
$$\left(a_1\right)\cdot\left(1-\lambda\right)\cdot\pi_{t-2}+s_t+\left(a_1\right)\cdot s_{t-1}+b\cdot a_2\cdot\mu_t+b\cdot a_3\cdot d_t \qquad \text{A10}$$

$$\pi_t=\frac{\left(1+a_1\right)\cdot\left(1-\lambda\right)}{\left(1-\lambda+b\cdot a_2\right)}\cdot\pi_{t-1}-\frac{\left(a_1\right)\cdot\left(1-\lambda\right)}{\left(1-\lambda+b\cdot a_2\right)}\cdot\pi_{t-2}+\frac{1}{\left(1-\lambda+b\cdot a_2\right)}\cdot s_t$$
$$+\frac{\left(a_1\right)}{\left(1-\lambda+b\cdot a_2\right)}\cdot s_{t-1}+\frac{b\cdot a_2}{\left(1-\lambda+b\cdot a_2\right)}\cdot\mu_t+\frac{b\cdot a_3}{\left(1-\lambda+b\cdot a_2\right)}\cdot d_t \qquad \text{A11}$$

$$\pi_t=\frac{1+a_1}{\left(1+\frac{b\cdot a_2}{1-\lambda}\right)}\cdot\pi_{t-1}-\frac{a_1}{\left(1+\frac{b\cdot a_2}{1-\lambda}\right)}\cdot\pi_{t-2}+\frac{1}{\left(1-\lambda+b\cdot a_2\right)}\cdot s_t$$
$$+\frac{\left(a_1\right)}{\left(1-\lambda+b\cdot a_2\right)}\cdot s_{t-1}+\frac{b\cdot a_2}{\left(1-\lambda+b\cdot a_2\right)}\cdot\mu_t+\frac{b\cdot a_3}{\left(1-\lambda+b\cdot a_2\right)}\cdot d_t \qquad \text{A12}$$

$$\pi_t=\alpha_1\cdot\pi_{t-1}-\alpha_2\cdot\pi_{t-2}+\gamma_1\cdot s_t+\gamma_2\cdot s_{t-1}+\gamma_3\cdot\mu_t+\gamma_4\cdot d_t \qquad \text{A13}$$

Where;

$$\gamma_1=\frac{1}{\left(1-\lambda+b\cdot a_2\right)} \qquad \gamma_2=\frac{a_1}{\left(1-\lambda+b\cdot a_2\right)}$$
$$\gamma_3=\frac{b\cdot a_2}{\left(1-\lambda+b\cdot a_2\right)} \qquad \gamma_4=\frac{b\cdot a_3}{\left(1-\lambda+b\cdot a_2\right)}$$

Equation A13 gives us an expression for the development over time of the inflation rate that takes both the AS and the AD curve into account. In ASAD.XLS equation A6 and A13 is used together with equation 13 to simulate the development over time for the inflation rate and the output gap.