Growth simulations in the Solow growth model

This document gives a description of the Solow growth model found at
http://www.natskolan.se/exercises/growth.htm

The purpose of the model is to give an increased understanding over the Solow growth model by simulating how the different variables develop over time.

The model simulates growth over a 90 year period and assumes that for the first 45 years a constant fraction of GDP is saved to finance investments. For the second 45 years another constant fraction is used so that we can study the effects from changes in the investments share. It is also possible to investigate the effect of population growth as well as technological progress.

Mathematical formulation of the model

The production function:

$$Y_t = A_t^{(1-\alpha)} K_t^{\alpha} L_t^{(1-\alpha)}$$

The production function is a Cobb Douglas with constant return to scale if both factors are increased but diminishing returns to one of the factors if the other is kept constant.

The Capital Stock is equal to last year capital stock less depreciation plus investments.

$$K_t = (1-\delta) K_{t-1} + I_{t-1}$$

Investments equal savings that is a fraction of GDP

$$I_t = sY_t$$

Labour force is growing with the population and will thus be equal to 1+*n* times last year labour force.

$$L_t = (1+n) L_{t-1}$$

Technological progress is exogenous and equal to *a*, meaning that the solow residual is equal to the Solow residual from last year times 1+*a*.

$$A_t = (1+a) A_{t-1}$$

Capital stock per worker and production per worker is equal to:

$$k = \frac{K}{L} \qquad y = \frac{Y}{L}$$

Capital stock per effective worker and production per effective worker is equal to:

$$\tilde{k} = \frac{K}{AL} = \frac{k}{A} \qquad \tilde{y} = \frac{Y}{AL} = \frac{y}{A}$$

Description of the model

The calculations is made in the excel file “solow.xls”. By changing the values in the yellow cells at the top of the sheet “growth over time” excel will produce graphs over the growth paths of the different variables in the figures below. At the same time a textbook figure is created in the sheet “textbook figure”and a graph over the development of capital stock investments and depreciations is created in the sheet “Invetsments and depreciations”.

In the B column row 2, 3 and 4 you should declare starting values for K, L and A respectively. In the D column you should declare populations growth depreciation rate and productivity growth in the rows 2, 3 and 4. In the cell F3 you should declare the saving fraction at the first 45 years and in G3 for the second 45 years. In cell F4 you should declare the value of α. In the blue cells you should not write anything. Here will the steady state values for the first and second time period be calculated automatically. Or more correctly, the values of year 45 and year 90, these values will be steady state values only if the model has converged to a steady state.

There are three different kinds of steady state values. The first will apply if we neither have population growth nor technological progress, the second will apply if we have population growth but no technological progress. If we have technological progress only the last kind of steady state values will apply.

The calculations are made automatically in the sheet “calculations”. Start with checking how these calculations are made. If you mark a cell the function in that cell will be shown in the function field.

The first column contains the years, the B column give you the value of A. in row 2 the start value is brought from the sheet “growth over time” in the rest of the column the value is calculated by multiplying the previous cell with 1+a. In column C the size of the labour force is calculated in a similar way.

In the D column the capital stock is calculated. From previous row the depreciations is subtracted and investments (also from the previous row but from column F) is added. Column E gives as the saving rates on the second and 47’th rows the values are brought from “growth over time” while the rest of the cells just takes the value from the row above. The saving rate will thus be constant in each of the two periods but the values can differ in the first and second period depending on what you declare in “growth over time”.

Investment is calculated in column F as the saving rate times production. Depreciations is calculated in column G by multiplying the depreciations rate with the capital stock. Production is calculated in column H using the production function. In column I the consumption is calculated as the production less investments. In the columns J, K and L the per worker variables is calculated by division with the labour force and in columns M and N the “per effective worker” variables are calculated by dividing column J and K with column B.